PHYSICS

1. $(F_{net})_{ext} = \mu_k(2m)g = (m_{total})a_{cm}$

$a_{cm} = \mu_k g$

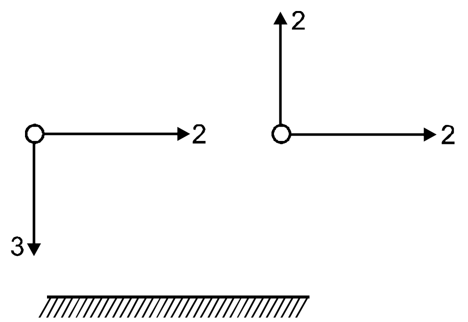
$S_{cm} = 0 + \frac{1}{2}(\mu_k g)t^2$

$S_{cm} = \frac{m_1S_1 + m_2S_2}{m_1 + m_2}$

$-\frac{1}{2}(\mu_k g)t^2 = \frac{(m)(x) + (m)(x - \ell)}{m + m}$

$x = \frac{\ell - (\mu_k g)t^2}{2}$.

2.



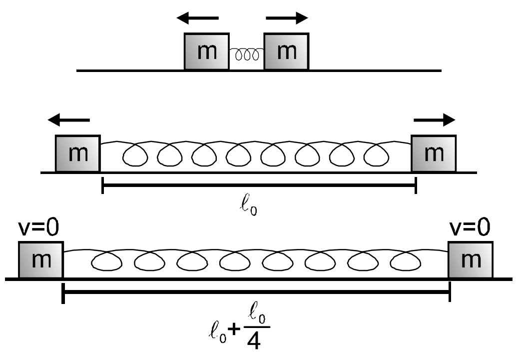
$e = \frac{v_{sep}}{v_{opp}} = \frac{2}{3}$.

3. As string does no work on the ball, energy conservation can be applied.

$$\frac{1}{2}mV^2 = mg\,(L - L\cos\theta) \Rightarrow \quad V = \sqrt{2gL(1-\cos\theta)}$$

on putting values $V = \sqrt{10}$ m/s

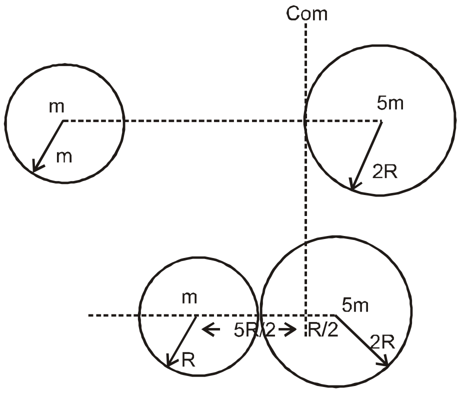
4.



Work energy theorem,

$$-\mu mg\left(\ell_0 + \frac{\ell_0}{4}\right) = \frac{1}{2}K\left(\frac{\ell_0}{4}\right)^2 - \frac{1}{2}K\ell_0^{\,2} \qquad \mu = \frac{3K\ell_0}{8mg}$$ **Ans.**

5.



Distance covered by the smaller sphere = $10R - \frac{5R}{2} = \frac{15R}{2}$

6. As $\Delta m << M$

So, we can assume that motion of mass M will not be influenced by Δm. Now, when total force on mass M is zero, let the compression in the spring is x.

by energy conservation

$$\Rightarrow kx = mg \Rightarrow x = \frac{mg}{k}$$

Now, maximum downwards displacement of M

$$2x = 2\frac{Mg}{k}$$

As block Δm is connected to mass M so its maximum upward displacement = $\frac{2Mg}{k}$ **Ans.**

7. $x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$

m_1 = mass of square plate

= m

x_1 = c. m. of square plate = 0

m_2 = mass of removed part

$$= -\frac{m}{\ell^2}\left(\frac{\pi\frac{\ell^2}{4}}{2}\right) = -\frac{\pi}{8}m$$

x_2 = c.m. of removed part

$$= \frac{\ell}{2} - \frac{4}{3\pi}\left(\frac{\ell}{2}\right) = \frac{\ell}{2}\left(1 - \frac{4}{3\pi}\right)$$

$$\therefore \quad x_{cm} = \frac{-\frac{\pi m}{8}.\frac{\ell}{2}\left(1-\frac{4}{3\pi}\right)}{m - \frac{\pi}{8}m} \qquad x_{cm} = -\frac{\ell\left(\pi - \frac{4}{3}\right)}{2\,(8-\pi)}$$

8. $P = Fv$

$v^2 = Fv$

$F = v$

$Ma = v$

$$1 \times v\frac{dv}{dx} = v$$

$$\int_1^v dv = \int_0^x dx$$

$v - 1 = x$

$v = x + 1$

$$\frac{dx}{dt} = x + 1$$

$$\int_0^x \frac{dx}{x+1} = \int_0^t dt$$

$\ell n(x + 1) - \ell n(0 + 1) = t$

$x + 1 = e^t$

$x = e^t - 1$

$x = e^t - 1$

at $t = \ell n2$

$x = 2 - 1 = 1$ m.

9. The horizontal component of velocity of sand just before falling on the cart is $v_s = 0$.

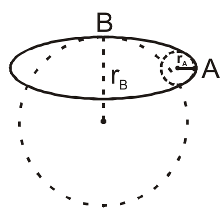
The horizontal speed of cart = v_C (constant).

The rate of mass falling on cart = μ (constant).

Horizontal force exerted by falling sand on cart = $\mu v_{rel} = \mu (v_c - v_s) = \mu v_c$

$\because$ μ and v_c are constant, the horizontal force is constant.

10. $$a_t = \frac{dv}{dt} = 0$$



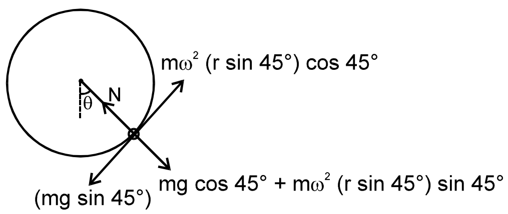
$$a_c = \frac{v^2}{R}$$

From A to B radius of curvature increases

So, acceleration decreases.

11. The maximum angular speed of the hoop corresponds to the situation when the bead is just about to slide upwards.

The free body diagram of the bead is



For the bead not to slide upwards.

$m\omega^2 (r \sin 45°) \cos 45° - mg \sin 45° < \mu N$ .... (1)

where $N = mg \cos 45° + m\omega^2 (r \sin 45°) \sin 45°$ .... (2)

From 1 and 2 we get.

$$\omega = \sqrt{30\sqrt{2}} \text{ rad / s.}$$

12. If velocity of m_2 is zero then
by momentum conservation

$m_1v' = m_2v$

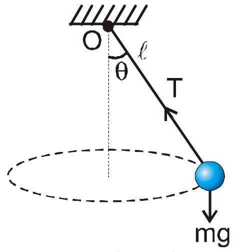
$$v' = \frac{m_2 v}{m_1}$$

Now kinetic energy of m_1

$$= \frac{1}{2}m^1v'^2 = \frac{1}{2}m_1\left(\frac{m_2}{m_1}\right)^2 v^2 = \frac{1}{2}\left(\frac{m_2}{m_1}\right)m_2v^2 = \left(\frac{m_2}{m_1}\right)\frac{1}{2}m_2v^2 = \frac{m_2}{m_1} \times \text{initial Kinetic energy}$$

Kinetic energy of m_1 > initial mechanical energy of system
Hence proved

13. For conical pendulum of length ℓ, mass m moving along horizontal circle as shown



$T\cos\theta = mg$ .... (1)

$T\sin\theta = m\omega^2\ell\sin\theta$ .... (2)

From equation (1) and equation (2), $\ell\cos\theta = \dfrac{g}{\omega^2}$

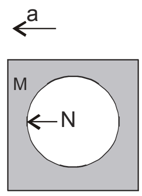
$\ell\cos\theta$ is the vertical distance of bob below O point of suspension. Hence if ω of all three pendulums are same, they shall revolve in same horizontal plane.

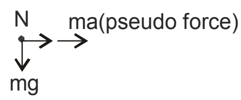
Alternate :

If we remember that time period T of conical pendulum is

$T = 2\pi\sqrt{\dfrac{L}{g}}$ where L is the vertical depth of bob below point of suspension. If T is same for three pendulums even L shall be also same. Hence all three particles shall revolve in same horizontal plane.

16. Let the normal force between the block and the ball be N.



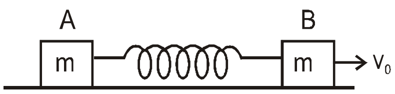


For the block, from Newton's II[nd] law , we have N = Ma = 2ma

For ball (with respect to the block), from Newton's II[nd] law , we have $N + ma = \dfrac{mv^2}{R}$

Solve the two equations.

17.



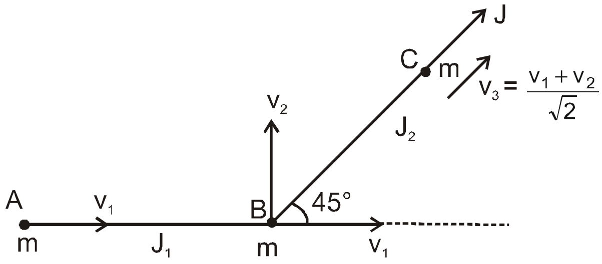
$1/2\, mv_0^2 = 1/2\, kx_0^2$

$v_{cm} = v_0/2 = \dfrac{1}{2}\sqrt{\dfrac{k}{m}}x_0, \quad 1/2\, kx^2_{max} = \dfrac{1}{2}\left(\dfrac{m}{2}\right)v_0^2$

$1/2\, kx^2_{max} = \dfrac{1}{2}\left(\dfrac{1}{2}kx_0^2\right) \quad \Rightarrow x_{max} = \dfrac{x_0}{\sqrt{2}}$

$(V_A)_{max} = (V_B)_{max} = v_0 = \sqrt{\dfrac{k}{m}}x_0$

18. $\frac{J_2}{\sqrt{2}} = 2mv_1$ $\frac{J_2}{\sqrt{2}} = mv_2$ $\Rightarrow$ $v_2 = 2\,v_1$



$J - J_2 = \frac{m}{\sqrt{2}}(v_1 + v_2)$

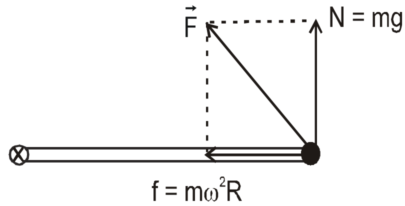
$J - 2\sqrt{2}\ mv_1 = \frac{3mv_1}{\sqrt{2}}$ $\Rightarrow J = \frac{7mv_1}{\sqrt{2}}$ $\Rightarrow v_1 = \frac{\sqrt{2}J}{7m}$

$v_2 = \frac{2\sqrt{2}J}{7m}$

$v_A = v_1 = \frac{\sqrt{2}J}{7m}$ $v_B = \frac{\sqrt{10}J}{7m}$

$v_c = \frac{v_1 + v_2}{\sqrt{2}} = \frac{3J}{7m}$.

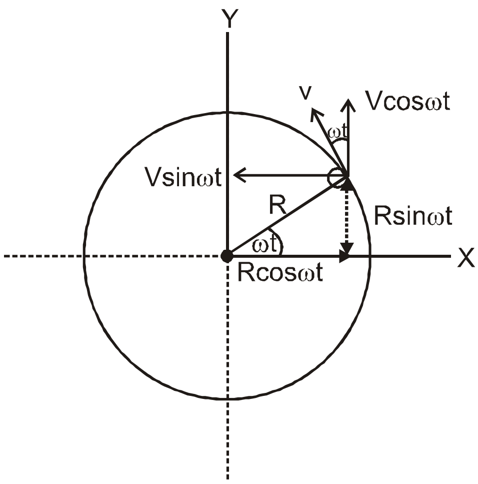
19. $F = \sqrt{f^2 + (mg)^2}$



Now when the angular speed of the rod is increasing at const. rate the resultant force will be more inclined towards $\vec{f}$.

Hence the angle between $\vec{F}$ and horizontal plane decreases so as with the rod.

20.



So X component of velocity $V_x = -V \sin\omega t$

y component of force $F_y = -mv^2/R \sin\omega t = -m\omega^2 R \sin\omega t$

Angular velocity of particle ω = constant.

X–coordinate of the particle $x = R\cos\omega t$. So B, C, D are correctly matched

21. $\vec{F} = \frac{\overrightarrow{dp}}{dt}$ = acost $\hat{i}$ + asint $\hat{j}$

$\vec{F}.\vec{P}$ = 0

magnitude of momentum :

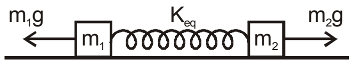
$= \sqrt{a^2\cos^2 t + a^2\sin^2 t} = a$

22. When speed of car is 40 km/hr, car can make a turn without skidding. If speed is less than 40 km/hr than tendency of slipping is downward so it will slip down. If speed is greater than 40 km/hr than tendency of slipping upward so it will slip up.

If the car's turn at correct speed 40 km/hr

$$N \sin\theta = \frac{mv^2}{r}, \qquad N = \sqrt{(mg)^2 + \left(\frac{mv^2}{r}\right)}$$ **Ans.**

23. Above situation can be represented as



Now at maximum elongation $v_{2/1} = 0$

Say at any moment elongation of spring is x

$a_{2/1} = 2g - k_{eq}x\left(\frac{1}{m_1} + \frac{1}{m_2}\right)$

$vdv = \left[2g - k_{eq}x\left(\frac{m_1 + m_2}{m_1 m_2}\right)\right]dx$

$\Rightarrow \qquad x_{max} = \frac{4m_1m_2g}{K_{eq}(m_1 + m_2)}$

Ans. $k_1x_1 = k_2x_2 = k_{eq}x$

24 $N = mg\cos\theta + m\omega^2R = mg\cos\theta + m\left(\frac{5g}{R}\right)R \Rightarrow N = 5\,mg + mg\cos\theta > 0$ so block does not leave circular motion

$f_r = ma_T$ For limiting case $\mu N = ma_T \qquad \Rightarrow \qquad \mu\,(mg\,(5 + \cos\theta)) = mg\sin\theta \Rightarrow \mu = \frac{\sin\theta}{5 + \cos\theta}$

$$\frac{d\mu}{d\theta} = \frac{(5+\cos\theta)(\cos\theta) - \sin\theta(-\sin\theta)}{(5+\cos\theta)^2} = 0 \Rightarrow \cos\theta = -\frac{1}{5}$$

$$\mu_{max.} = \frac{\sqrt{1 - \frac{1}{25}}}{5 - \frac{1}{5}} = \frac{2\sqrt{6}}{24} \Rightarrow x = 8$$

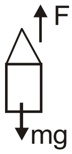
25. Force on table due to collision of balls :

$$F_{dynamic} = \frac{dp}{dt} = 2 \times 20 \times 20 \times 10^{-3} \times 5 \times 0.5 = 2\text{ N}$$

Net force on one leg $= \frac{1}{4}(2 + 0.2 \times 10) = 1$ N

26. Thrust force

$F = \frac{dm}{dt} . u_{rel} = 20 \times 1000$ N

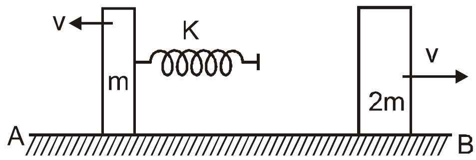


$F_{net} = F - mg$
$= 20000 - 10000$
$= 10000$ N

$\therefore \quad a = 10\ m/s^2$

$\therefore \quad \frac{a}{g} = 1$

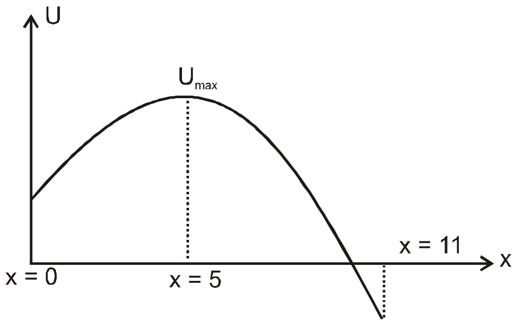
27. Situation after long time



Work done = $\Delta K = \frac{1}{2}(2m)(v)^2 - \frac{1}{2}(2m)(3v)^2 =$

$= mv^2 - 9mv^2 = -8mv^2$.

28. Draw U v/s x graph. There is a maxima of potential energy between x = 11 to x = 0. So to bring the particle from x = 11 to x = 0, the particle has to cross the maxima (x = 5) and to just cross the point x = 5, velocity at x = 5 should be 0^+.



$\Rightarrow$ Applying energy conservation between x = 11 to x = 5.

$k_i + U_i = k_f + U_f$

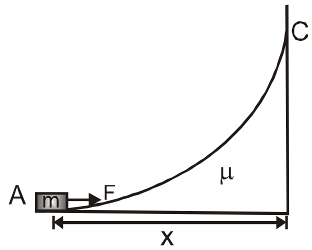
$\frac{1}{2}(0.5)\,u^2 + (30 - (11-5)^2) = 0^+ + (30 - (5-5)^2)$

u = 12 m/sec $\Rightarrow$ $\frac{u}{2} = 6$ m/sec

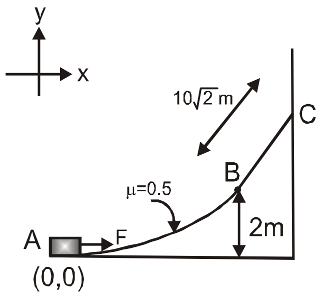
29. Since angular velocities of the particles are different, after some time, two particles may move parallel. In such case $\left|\vec{P}_A + \vec{P}_B\right|$ is maximum.

$\left|\vec{P}_A + \vec{P}_B\right|_{max} = (2 \times 2 + 1 \times 3)$ kg m/s = 7 kg m/s

30. Slope of line BC = $\frac{dy}{dx} = \frac{2x}{8} = \frac{2 \times 4}{8} = 1 \quad \Rightarrow \quad \theta = 45^\circ$



If the mass m is taken from A to C slowly work done by friction will always be equal to the $W_f = -\mu mgx$



Now, by $\quad W_{net} = \Delta KE = 0$

$W_F - mg(10 + 2) - \mu mg(10 + 4) = 0$

$\Rightarrow \quad W_F = 380 = 76 \times 5 \quad \Rightarrow \quad \lambda = 5.$

31. $a_t = g \sin 60^\circ = \frac{\sqrt{3}g}{2}$

$a_c = \frac{v^2}{R}$

$\frac{1}{2} mv^2 - mgR \cos 60 = \frac{1}{2} m(4gR) - mgR.$

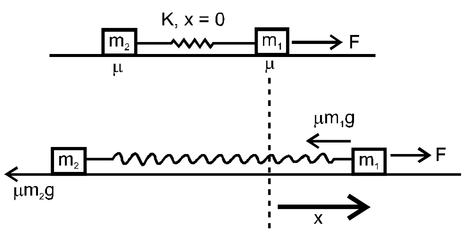
$\frac{1}{2} mv^2 = \frac{3}{2} mgR$

$v^2 = 3gR$.

$a_c = \frac{3gR}{R} = 3g, \quad a_t = \frac{\sqrt{3}g}{2}, \; a_c = 3g$

$P = \frac{a_c}{a_t} = \frac{2 \times 3g}{\sqrt{3}g} = 2\sqrt{3}$ **Ans.**

32.

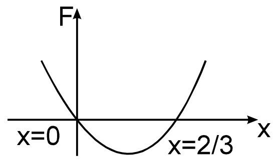


WF + WSp + Wfric = ΔK

$\Rightarrow \quad Fx - \frac{1}{2} Kx^2 - \mu m_1 g x = 0 \quad$ and $\quad Kx = \mu m_2 g$

$\Rightarrow \quad F - \frac{1}{2} \mu m_2 g - \mu m_1 g = 0 \quad \Rightarrow \quad F = \mu m_1 g + \frac{\mu m_2 g}{2} = 0.1 \times 10 \times 10 + \frac{0.1 \times 20 \times 10}{2} = 20$ N

33. The particle is at equilibrium at x = 0 and x = $\frac{2}{3}$.



The minimum speed imparted to the particle should be such that it just reaches x = $\frac{2}{3}$ from there on it shall automatically reach x = 0

$$\frac{1}{2}mv^2 = -\int_4^{2/3} F\,dx = -\int_4^{2/3} x(3x-2)dx = \frac{1300}{27} \quad \text{or} \quad v = \sqrt{\frac{2600}{27}}\ \text{m/s}$$

34. $MX = m(R - R\cos\theta - X] \Rightarrow \quad X = \frac{mR(1-\cos\theta)}{(M+m)}$

35. Momentum of system in horizontal will remain conserved, so

$$MV = m(v\sin\theta - V)$$

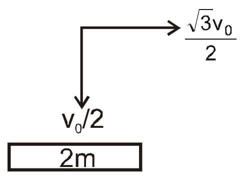
$$\Rightarrow \quad V = \frac{mv}{M+m}\sin\theta$$

$$\frac{dV}{dt} = \frac{mv}{M+m}\cos\theta\left(\frac{d\theta}{dt}\right)$$

$$\left(\frac{dV}{dt}\right) = \left(\frac{m}{M+m}\right)\frac{v^2}{R}\cos\theta$$

37 to 39.

JBC



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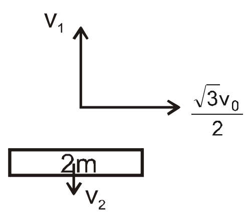


$v_1 + v_2 = V_0/2$

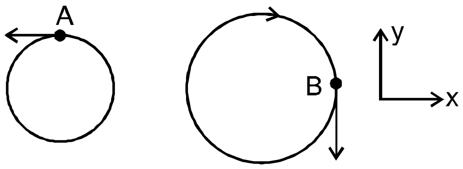
$2mv_2 - mv_1 = m\frac{v_0}{2} \quad \Rightarrow \quad v_1 = \frac{v_0}{6} \qquad v_2 = \frac{v_0}{3}$

Maximum compression $= \frac{v_2}{\sqrt{\frac{K}{2m}}} = \sqrt{\frac{2m}{k}}\frac{v_0}{3}$

Maximum height $= \frac{v_1^2}{2g} = \frac{v_0^2}{72g}$

minimum kinetic energy $= \frac{1}{2}m\left(\frac{\sqrt{3}v_0}{2}\right)^2 = \frac{3mv_0^2}{8}$

40. At time t = 1 sec positions of A and B are



acceleration of A $\vec{a}_A = \omega_1^2 r_1(-\hat{j})$ = $\left(\frac{\pi}{2}\right)^2 (1)(-\hat{j})$

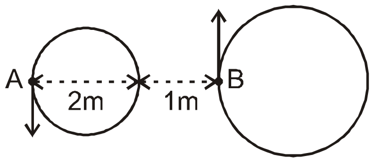
41. At time t = 1 sec

$\vec{a}_B = \omega_2^2 r_2 (-\hat{i})$ = $2\pi^2(-\hat{i})$

$\vec{a}_A - \vec{a}_B = \frac{\pi^2}{4}(-\hat{j}) + 2\pi^2(\hat{i})$

a_{rel} = $\pi^2\left[\frac{1}{16}+4\right]^{1/2}$ = $\frac{\pi^2}{4}\sqrt{65}$ m/sec^2

42. At time t = 2 sec, position of A and B are



$v_A = \omega_1 r_1 = \frac{\pi}{2}(1) = \frac{\pi}{2}$ m/sec.

$v_B = \omega_2 r_2 = 2\pi$ m/sec.

distanceAB = 3m

$\omega = \frac{v_A + v_B}{AB} = \frac{\pi/2 + 2\pi}{3} = \frac{5\pi}{6}$ rad/sec.

43. (P) Since external force in horizontal direction is zero there for COM remains at rest.
(Q) If the block remains at rest then centre of mass moves with constant velocity.
(R) If m does not slips on M then COM remains at rest otherwise COM is accelerated when m moves from point A to B.
(S) The COM is accelerated vertically downwards by the gravity force.

45. Ball only loose contact with surface B when v is in range $\sqrt{2Rg} < v < \sqrt{5Rg}$ so for A,B,D maximum value of N_A is zero for option C ball lose contact with surface B at some point.
maximum value of N_B is lowest point and given

$$N = mg + \frac{mv^2}{R}.$$